Combination of Spectral and SVD Precodings for Out-of-Band Leakage Suppression

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***Abstract*—Since the out-of-band (OOB) power suppression of Orthogonal Frequency Division Multiplexing (OFDM) has been an essential topic for various wireless communication applications such as Cognitive Radio (CR) systems, in this paper, two existing precoding techniques are investigated. Then a combined approach which properly adjusts the matched code rates of these two investigated techniques is proposed. Numerical results show that, for OFDM without guard intervals (NG-OFDM) and with zero padding (ZP-OFDM), the proposed method gives significant further improvement on OOB power suppression and almost no extra drawback than either of the two original techniques.**

***Index Terms*—CR, OFDM, OOB power suppression, precoding.**

I. Introduction

Cognitive Radio (CR), as a promising solution to the spectrum congestion problem brought by the rapid increasing number of wireless communication techniques and devices, has drawn significant attention recently [1], [2]. A CR system is required to be able to operate in the bands assigned to licensed users (LUs) by utilizing vacant parts of LU bands and minimize its interference to LUs. Orthogonal frequency division multiplexing (OFDM) technology, which divides the total bandwidth into several orthogonal sub-bands overlapping in frequency, has been considered as a candidate for CR in the first cognitive radio based standard IEEE 802.22 [3]. OFDM has several favorable properties like high spectral efficiency, robustness to channel fading, multipath delay spread tolerance, efficient fast Fourier transform (FFT) implementation, etc. [4]. However, one of the major problems is the high sidelobes of all its subcarriers. As a result, OFDM based CR systems suffer from large out-of band (OOB) radiation that interferes with other bands occupied by LUs.

Several schemes have been proposed to reduce the OOB radiation of OFDM based CR systems. A straightforward way is to use filtering or windowing techniques [5-7], but this would introduce degradation of bit error rate (BER) performance. Another method, which is to disable some of the CR subcarriers to create some guard bands between CR bands and LU bands, may not be efficient because we may need to use a large number of guard bands to reduce the interference to a practically acceptable level. A similar method called Cancellation Carriers (CC) is proposed in [8]. Instead of just disabling subcarriers, the inputs of these subcarriers are designed so that the radiation at certain frequencies, which are usually assigned to LUs, is minimized. The main drawback is that the design of the inputs depends on the inputs of the remaining data subcarriers, which causes a very large computational complexity. So does the Subcarrier Weighting (SW) method proposed in [9]. SW can be essentially viewed as a precoding method with a real diagonal matrix that does not decrease the spectral efficiency. A more sophisticated precoding method which does decrease the spectral efficiency is proposed in [10]. A precoding matrix of a less-than-one code rate is designed to reduce the OOB radiation. In other word, it cannot be a square matrix as in SW. This matrix design, however, does not depend on the input data. Thus the complexity is drastically decreased. This method will be called Beek's singular value decomposition (SVD) precoding in this paper due to the principle of its design. Another precoding scheme called Chung's spectral precoding, which is also independent of the input data, is proposed in [11]. Instead of SVD precodings that minimize the system's energy at certain frequencies, [11] uses new orthogonal basis sets to replace the rectangular pulse for each conventional OFDM symbol so that the new sidelobes fall off faster than those of the sinc functions. The spectral efficiency is also reduced due to the limited number of available basis sets when the in-band range is fixed. It's seen, however, the most significant OOB power suppression improvement happens when the spectral efficiency is reduced from 1 to $(N-1)/N$ and from $(N-1)/N$ to $(N-2)/N$, where N is the number of subcarriers. As the spectral efficiency continues to decrease, the improvement becomes not as significant as it was. Based on this observation, an assumption then comes that if we redistribute part of the total spectral efficiency loss from Chung's spectral precoding to some other precoding schemes such as SVD precoding schemes, the resulting combined schemes might have better OOB power suppression effects than either of the schemes used separately.

Therefore in this paper, we combine Beek's SVD precoding

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in [10] with Chung's spectral precoding in [11] since each technique designs a precoding matrix which can be conveniently multiplied with the other. By properly adjusting the two matrices' dimensions, i.e., by properly assigning the matched code rates, we could make the optimum combined new matrix have better OOB power suppression effect than either of the two original matrices.

The paper is organized as follows: Section II describes the OFDM system model. Section III briefly introduces the two existing precoding schemes. Section IV presents the proposed combined scheme. Section V presents the numerical results including power spectral density (PSD), peak-to-average power ratio (PAPR) and BER results for different scenarios. Finally, the conclusion is in Section VI. Notations are as follows. All boldface letters indicate vectors (lower case) or matrices (upper case). Matrix $\mathbf{I}$ means an identity matrix of proper dimension. $\mathbf{A}^H$ stands for the conjugate transpose of $\mathbf{A}$. $a_{i,j}$ denotes the $(i,j)^{\text{th}}$ entry of $\mathbf{A}$.

II. System Description

The block diagram of a typical OFDM system adopting a precoding technique is shown in Fig. 1. As shown in the figure, the source bit stream is first mapped into a symbol stream by PSK/QAM modulation and goes through the serial-to-parallel (S/P) conversion. Let $\mathbf{d}_l = [d_{1,l} \quad d_{2,l} \quad \dots \quad d_{K,l}]^T$ denote the l^{th} data vector, where l is the index in the time domain and K is the length of each vector. Then each vector is left-multiplied by an $N \times K$ precoding matrix $\mathbf{G}$, i.e.,

$$\mathbf{b}_l = \mathbf{G}\mathbf{d}_l \tag{1}$$

where $\mathbf{b}_l = [b_{1,l} \quad b_{2,l} \quad \dots \quad b_{N,l}]^T$ denotes the l^{th} precoded vector. The code rate is defined as K/N, which is obviously no larger than 1. $\mathbf{x}_l$ is the Inverse Fast Fourier Transform (IFFT) output of $\mathbf{b}_l$. Cyclic Prefix (CP) or Zero Padding (ZP) could be added to $\mathbf{x}_l$ when we need to do so to counteract the channel effects. At the receiver, CP or ZP is removed from the received vector $\mathbf{r}_l'$. Then it goes through the FFT block, decoded by being left-multiplied by a $K \times N$ decoding matrix $\widehat{\mathbf{G}}$, i.e.,

$$\hat{\mathbf{d}}_l = \widehat{\mathbf{G}}\hat{\mathbf{b}}_l \tag{2}$$

If the channel is ideal, then the data would be correctly decoded only if

$$\widehat{\mathbf{G}}\mathbf{G} = \mathbf{I} \tag{3}$$

III. Existing Precoding Schemes

A. Beek's SVD precoding [10]

In [10], the continuous time-domain transmit signal $x_l(t)$ for a given $\mathbf{b}_l$ in a CP-OFDM system is expressed as

$$x_l(t) = \sum_{i=1}^{N} b_{i,l} p_i(t) \tag{4}$$

where $p_i(t)$ is the windowed subcarrier waveform:

$$p_i(t) = e^{j2\pi\frac{i}{T_d}t} g_c(t) \tag{5}$$

with the pulse shape function

$$g_c(t) = \begin{cases} 1, -T_{CP} \le t < T_d \\ 0, otherwise. \end{cases} \tag{6}$$

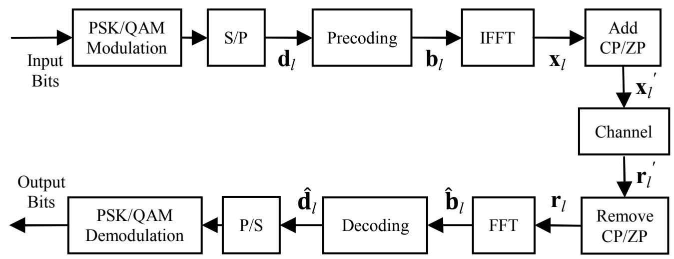

Fig. 1. Block diagram of OFDM system using precoding technique

In (6), T_d is the effective symbol duration and T_{CP} is the cyclic prefix duration.

Note that the frequency domain of $x_l(t)$ is

$$X_l(f) = \sum_{i=1}^{N} b_{i,l} P_i(f) \tag{7}$$

where

$$P_i(f) = \frac{e^{j\pi\left(\frac{i}{T_d}-f\right)(T_d-T_{CP})}}{\pi\left(\frac{i}{T_d}-f\right)T} \sin\left(\pi\left(\frac{i}{T_d}-f\right)T\right) \tag{8}$$

where $T = T_{CP} + T_d$ is the entire symbol duration. Note that $P_i(f)$ in a ZP-OFDM system could be derived similarly.

Assume that we want to minimize the radiation power at the frequencies $f_1, f_2, \dots, f_M$ by designing a precoding matrix $\mathbf{G}^S$. Denote $\mathbf{X}_l = [X_l(f_1) \quad X_l(f_2) \quad \dots \quad X_l(f_M)]^T$, we have

$$\mathbf{X}_l = \mathbf{P}\mathbf{G}^S\mathbf{d}_l\,, \mathbf{P} = \begin{bmatrix} P_1(f_1) & \cdots & P_N(f_1) \\ \vdots & \ddots & \vdots \\ P_1(f_M) & \cdots & P_N(f_M) \end{bmatrix} \tag{9}$$

To minimize $\|\mathbf{X}_l\|$ regardless of $\mathbf{d}_l$, perform the SVD of $\mathbf{P}$ that factorizes $\mathbf{P}$ as

$$\mathbf{P} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{10}$$

where $\mathbf{U}$ is an $M \times M$ unitary matrix, $\mathbf{\Sigma}$ is a diagonal $M \times N$ matrix containing the singular values of $\mathbf{P}$ in non-increasing order and $\mathbf{V}$ is an $N \times N$ unitary matrix whose columns are $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_N$. The precoding matrix is then chosen as:

$$\mathbf{G}^S = [\mathbf{v}_{N-K+1} \quad \mathbf{v}_{N-K+2} \quad \cdots \quad \mathbf{v}_N] \tag{11}$$

Define $R = N - K$ as the coding redundancy. If $R \ge M$, then $\|\mathbf{X}_l\| = 0$ for any arbitrary $\mathbf{d}_l$ because $\mathbf{b}_l$ is always in the null space of $\mathbf{P}$.

B. Chung's spectral precoding [11]

It is pointed out in [11] that the rectangularly pulsed OFDM possesses discontinuous pulse edges and exhibits relatively large power spectral sidelobes which fall off as f^{-2}. While the continuous-phase OFDM signals exhibit relatively small power spectral sidelobes which fall off as f^{-4}, and can thus provide much higher spectral efficiency than the traditional rectangularly pulsed OFDM signal. Two families of new basis sets which satisfy the continuous-phase requirement are introduced, named as family W and family V, respectively.

The corresponding precoded OFDM structure that is used to construct OFDM signals using the basis sets along with the arbitrary input data is given below. The entries of the family W_L-based precoding matrix $\mathbf{G}^{W_L}$ is defined as

$$\begin{gathered} g^{W_L}_{N(1-2^{1-u})+n,n2^u+v} = 2^{-\frac{u}{2}}(-1)^{1+\psi_{u,v}}, \\ n \in \left[0, \frac{N}{2^u} - 1\right] \text{ and } v \in [0, 2^u - 1], \end{gathered} \tag{12}$$

for $u = 1,2,\dots,L$. In (12), $\psi_{u,v}$ is the sum of the most and least significant bits in the binary representation (in bits) of the modulo-2^u value of v when $u \geq 2$ and $\psi_{1,v} = 1$ by default. All other entries equal 0.

The entries of the family V_L-based precoding matrix $\mathbf{G}^{V_L}$ is defined as

$$g^{V_L}_{N(1-2^{1-u})+n,n+\frac{N}{2^u}v} = 2^{-\frac{u}{2}}\phi_{u,v},$$
$$n \in \left[0, \frac{N}{2^u} - 1\right] \text{ and } v \in [0, 2^u - 1], \quad (13)$$

for $u = 1,2,\dots,L$. In (13), $\phi_{u,v} = 1$ if $u = log_2 N$ and $\phi_{u,v} = (-1)^{\zeta_v}$ otherwise, where ζ_v represents the least significant bit in the binary representation of v. All other entries equal 0.

L in (12) and (13) is a parameter that determines the code rate, which equals $1 - 2^{-L}, L \in [1, log_2 N]$. Since $\mathbf{G}^S$, $\mathbf{G}^{W_L}$ and $\mathbf{G}^{V_L}$ are all left unitary matrices containing orthonormal columns, then the decoding matrices are simply their conjugate transposes.

IV. Combined Precoding Design

We can combine Beek's SVD precoding matrix and Chung's spectral precoding matrix (using $\mathbf{G}^{W_L}$ as example) by defining either $\mathbf{G} = \mathbf{G}^S\mathbf{G}^{W_L}$or $\mathbf{G} = \mathbf{G}^{W_L}\mathbf{G}^S$. The former way surely is not good because the continuous-phase property of $\mathbf{G}^{W_L}\mathbf{b}_l$ cannot be maintained after it's left-multiplied by $\mathbf{G}^S$, which means $\mathbf{G}^{W_L}$ would be useless. On the other hand, by choosing the latter way, we can keep not only the continuous-phase property, but also the advantage of Beek's SVD precoding.

Assume that the code rates for both schemes have been determined. Here are the steps of the combined design. First, using (12), design

$$\overline{\mathbf{G}}^{W_L} = \mathbf{G}^{W_L} \quad (14)$$

without considering $\mathbf{G}^S$. Second, perform SVD of $\mathbf{Q} = \mathbf{P}\mathbf{G}^{W_L}$, i.e., let

$$\mathbf{Q} = \mathbf{P}\mathbf{G}^{W_L} = \overline{\mathbf{U}}\overline{\mathbf{\Sigma}}\overline{\mathbf{V}}^H. \quad (15)$$

Next, using (11) and (14), we have

$$\overline{\mathbf{G}}^S = [\overline{\mathbf{v}}_{N-K+1} \quad \overline{\mathbf{v}}_{N-K+2} \quad \dots \quad \overline{\mathbf{v}}_N]. \quad (16)$$

Finally, let

$$\mathbf{G} = \overline{\mathbf{G}}^{W_L}\overline{\mathbf{G}}^S \text{ and } \widehat{\mathbf{G}} = (\overline{\mathbf{G}}^{W_L}\overline{\mathbf{G}}^S)^H. \quad (17)$$

Note that the transmitted signal $\mathbf{x}_l$ for Chung's spectral precoding in [11] is the real part of the IFFT output, while the complex output is used for Beek's SVD precoding in [10]. In this paper, as shown in Fig. 1, the complex output is used.

The remaining problem is how we distribute the matched dimensions of $\mathbf{G}^S$ and $\mathbf{G}^{W_L}$ for a given dimension of $\mathbf{G}$ so that we can achieve the best OOB power suppression effect. The proposed distribution will be shown in next section.

V. Numerical Results

First, we consider a 64-subcarrier OFDM system without any CP or ZP. QPSK modulation is used. The FFT size is 256.

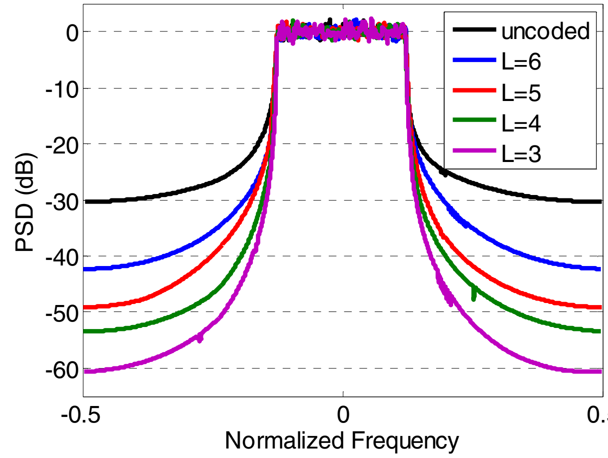

Fig. 2. PSD of W_L-based systems

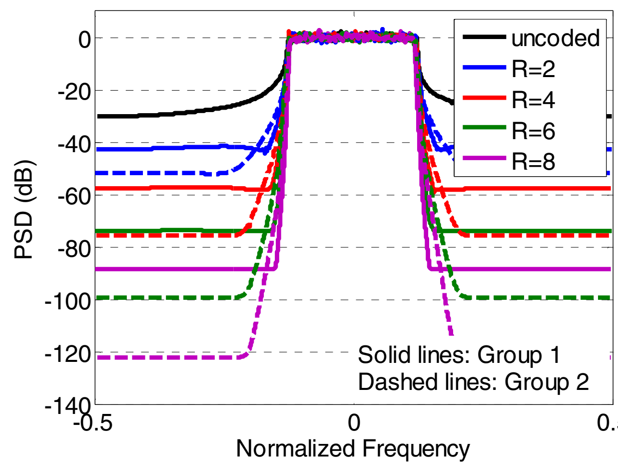

Fig. 3. PSD of SVD-based systems

Fig. 2 shows the PSD after applying Chung's spectral precoding only. Our simulation results show that the PSD of every W_L-based system outperforms that of the V_L-based system with the same L for every scenario (NG/ZP/CP-OFDM), which is in accord with the numerical results presented in [11]. Therefore, in this paper we only present figures of the W_L-based systems here and the combined SVD plus W_L-based systems in other figures. Note that the code rate is $1 - 2^{-L}$, which means when L is 6, 5, 4 or 3, the code rate is 63/64, 62/64, 60/64 or 56/64. The five curves show that the largest OOB power decrement appears when the code rate drops from 1 (uncoded) to 63/64 (L=6). As L further decreases linearly, the cumulative suppression impact becomes less and less effective as the code rate drops exponentially.

Fig. 3 shows the PSD after applying Beek's SVD precoding only. We use two groups of the notched frequencies, which are group1 (close notched frequencies) with$[-14.5\ -13.5\ -12.5\ -11.5\ \ 74.5\ \ 75.5\ \ 76.5\ \ 77.5$ and group 2 (distant notched frequencies) with $[-35.5\ -34.5\ -33.5\ -32.5\ \ 95.5\ \ 96.5\ \ 97.5\ \ 98.5$.

Considering that the active subcarrier index is 0-63 and the FFT size is 256, we can say that group 1 is close to the in-band and group 2 is distant. Fig. 3 shows that the allocation of the chosen notched frequencies indeed provides a tradeoff between the OOB power and the decaying rate. The power

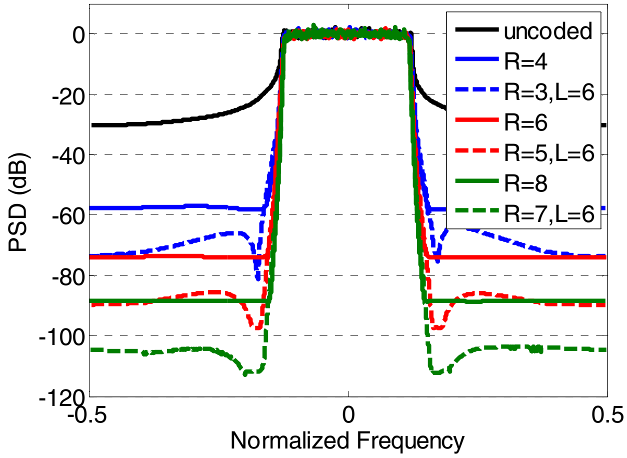

Fig. 4(a). SVD-based and combined NG-OFDM, group 1 notched frequencies

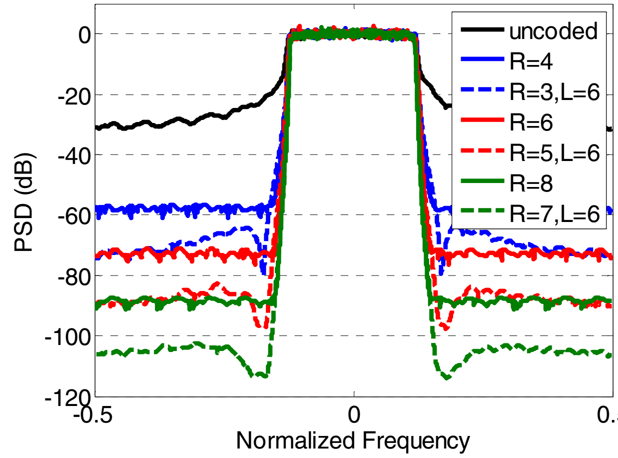

Fig. 5(a). SVD-based and combined ZP-OFDM, group 1 notched frequencies

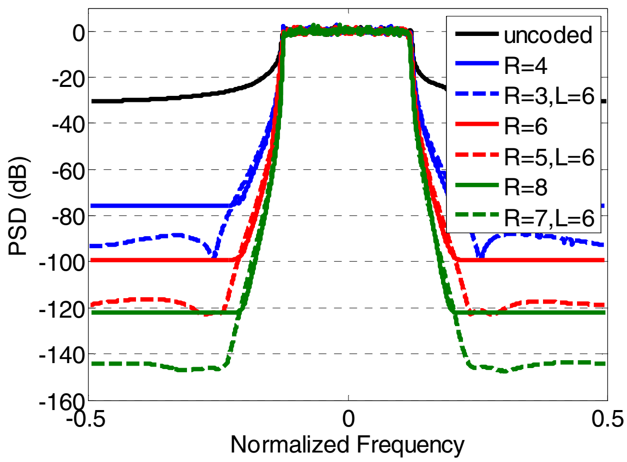

Fig. 4(b). SVD-based and combined NG-OFDM, group 2 notched frequencies

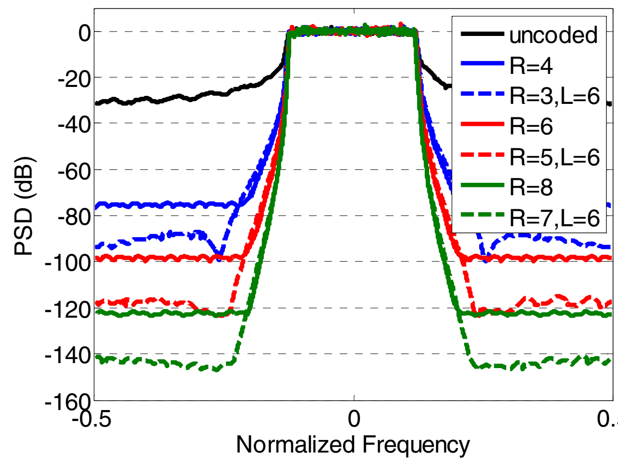

Fig. 5(b). SVD-based and combined ZP-OFDM, group 2 notched frequencies

decrement per 1/64 coding decrement approximately does not change. And it is much larger than that of Chung's spectral precoding when code rate is less than 62/64, compared with Fig. 2. Based on this observation, we assume that when the overall code rate is fixed as K/N and $K < N-2$, the best OOB power suppression effect is achieved by adopting the $[K/(N-1), (N-1)/N]$ code rate pair, which means assigning $K/(N-1)$ as the Beek's SVD precode rate and $(N-1)/N$ as the Chung's spectral precode rate. This assumption is partially verified by testing all possible code rate pairs for N=64,128. Due to the page limit, in the next few figures, we only present $[K/(N-1), (N-1)/N]$ with N=64 (i.e., R=63-K, L=6) to compare with using Beek's SVD precoding only (i.e., R=64-K) for three different scenarios, including NG-OFDM, ZP-OFDM and CP-OFDM.

Figs. 4 (a & b) show the comparison for NG-OFDM. Every two lines with the same color have the same corresponding overall code rates. Fig. 4(a) is based on the notched frequencies group 1 in Beek's SVD precoding and combined schemes, and Fig. 4(b) is based on the group 2. The same rule applies to Figs. 5-6. From Fig. 4(a) we see that, for all three code rates, the combined scheme gives about 15dB lower total OOB power than the Beek's SVD precoding scheme at the cost of a slightly wider transition band which is hardly observable in the figure. The transition band difference between the SVD and combined schemes is larger in Fig. 4(b), but the relatively significant OOB power decrement is seen by using the combined scheme.

Figs. 5 (a & b) show the comparison for ZP-OFDM. Here the length of ZP is $T_d/16$, i.e., 1/16 of the data block's length. The figures look almost the same as Figs. 4 because: 1) the continuous-phase property of Chung's spectral precoding is maintained since the value on both edges of each data block before ZP is also zero and 2) the right singular vectors of $\mathbf{P}$ in a ZP-OFDM, unlike CP-OFDM, are the same as NG-OFDM even though zeros are added at the beginning of each data block.

Figs. 6 (a & b) show the comparison for CP-OFDM. Here the length of CP is also $T_d/16$. Since CP is added and the starting edge of CP is usually not zero, then the Chung's spectral precoding scheme cannot construct continuous signal with CP. Therefore, assigning $1/K$ of the total spectral efficiency loss to Chung's spectral precoding and the rest $(K-1)/K$ to Beek's SVD precoding may not be better than assigning all the spectral efficiency loss to Beek's SVD precoding only. Furthermore, by comparing Figs. 6 to Figs. 4-5, we see that the OOB power suppression effects of all three code rates for CP-OFDM are much worse than NG-OFDM

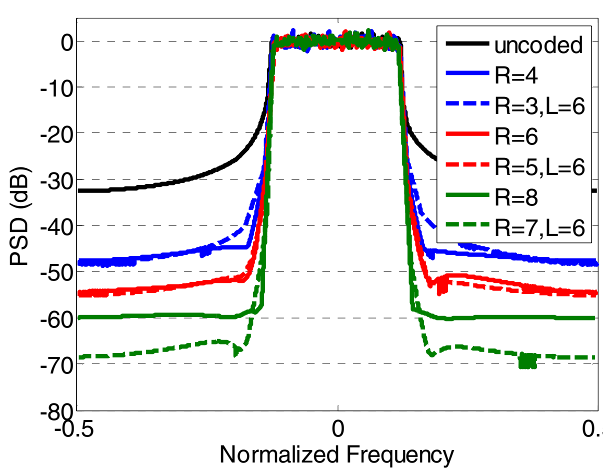

Fig. 6(a). SVD-based and combined CP-OFDM, group 1 notched frequencies

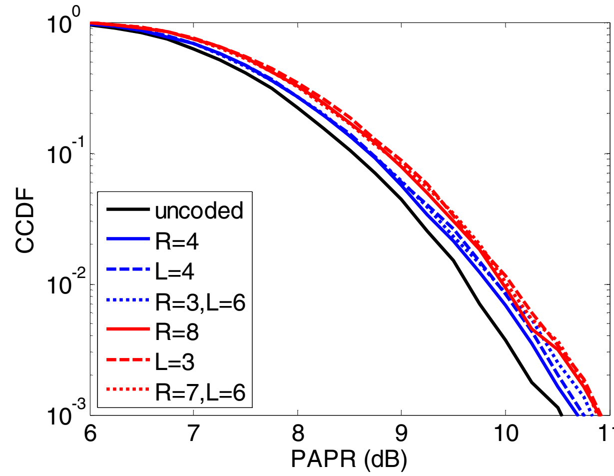

Fig.7. CCDF comparison

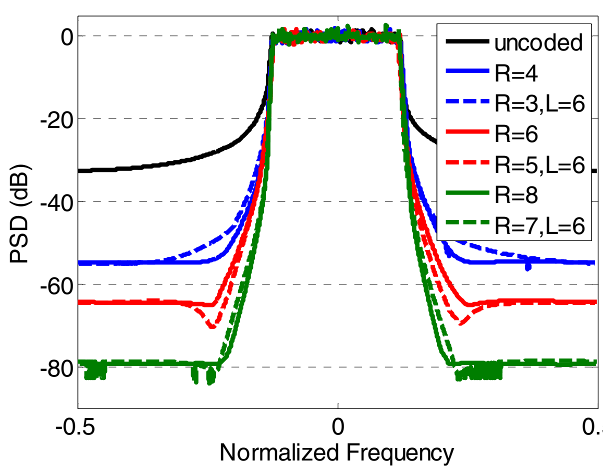

Fig. 6(b). SVD-based and combined CP-OFDM, group 2 notched frequencies

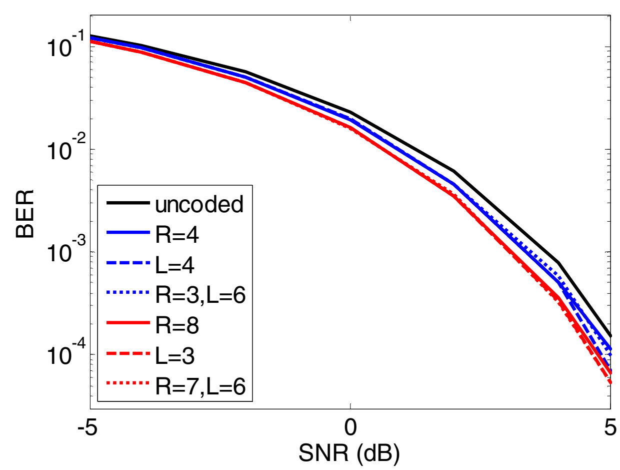

Fig.8. BER comparison for AWGN channel

and ZP-OFDM. This is because in NG/ZP-OFDM, the width of the sidelobes is equal to the frequency spacing of adjacent subcarriers, which means each sidelobe of one subcarrier completely overlaps with some sidelobes from all the other subcarriers. As a result, the singular values of $\mathbf{P}$ in (10) drop quickly. However, when CP is added and symbol duration is increased, the width of the sidelobe becomes narrower. Therefore, the singular values of $\mathbf{P}$ drop more slowly. Let P_I be the average power leakage after precoding at the chosen notched frequencies f_l [12]. It can then be expressed by

$$P_I = P_s \sum_{i=R}^{N-1} \sigma_i^2(\mathbf{P}) \tag{18}$$

where P_s is the average power of $\mathbf{d}_l$ and $\sigma^i(\mathbf{P})$ is the i^{th} largest singular value of $\mathbf{P}$. In this sense, for the same value of R, the power leakage P_I of CP-OFDM is larger than the P_I of NG-OFDM or ZP-OFDM.

Next, we check the PAPR performance of the combined scheme. Fig. 7 shows the complementary cumulative distribution function (CCDF) of the three schemes' IFFT outputs in an NG-OFDM system using QPSK modulation. The number of subcarrier is 64 and FFT size is 256. Each curve of the same color has the same code rate. We can see that the code rate is the decisive factor of the PAPR distribution. The "uncoded" (code rate is 1) has the best performance while the red curves (code rate is 56/64) have the worse. When the code rate remains unchanged, the Beek's SVD precoding scheme (solid curves) looks slightly better. But such a subtle difference might be due to the insufficient number of collected samples and is nearly negligible compared with the difference brought by the change of the overall code rate.

Finally, we check the BER performance. Here the channel is assumed as an AWGN channel, which means $\mathbf{r}_l' = \mathbf{x}_l' + \mathbf{n}_l$, where $\mathbf{n}_l$ denotes the noise vector. SNR (dB) is defined as

$$SNR = 10\, log_{10} \frac{E\left(|\mathbf{x}_l'|^2\right)}{E(|\mathbf{n}_l|^2)} \tag{19}$$

Fig. 8 shows the BER of the same system used for Fig. 7. Each curve of the same color has the same code rate. The combined scheme, which uses the distant notched frequencies, has almost the same BER curves as Beek's SVD precoding or Chung's spectral precoding when they have the same code rates. Furthermore, as the code rate decreases, the BER becomes slightly better (smaller) because the length of $\mathbf{d}_l$, which needs to be estimated at the receiver, is reduced while the row size of the precoding matrix is unchanged. That means that a lower code rate results in more reliable transmission (i.e., lower BER at same SNR, or less transmit power to achieve same BER).

VI. CONCLUSION

In this paper, we first investigated two different precodeing schemes to reduce the OOB radiation of OFDM systems, namely Beek's SVD precoding [10] and Chung's spectral precoding [11]. Then we proposed a combined precoding scheme by redistributing the overall K/N code rate of the system into a $[K/(N-1), (N-1)/N]$ code rate pair and then assigning $K/(N-1)$ to Beek's SVD precoding and $(N-1)/N$ to Chung's spectral precoding sequentially. Numerical results show that the combined scheme can further reduce the OOB power by more than 15dB than either of the two original schemes for NG-OFDM and ZP-OFDM systems regardless of how we choose the notched frequencies in Beek's SVD precoding. The PAPR performance of the combined scheme is almost the same as the two isolated schemes because the overall code rate has the biggest effect on PAPR. And when the AWGN channel is considered, the BER performance of the combined scheme, which is slightly better than the uncoded system at the cost of the reduced code rate, is also the same as the other two schemes.

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