

# EXPERIMENTAL AND THEORETICAL STUDY OF SHORT-TERM SIGNAL VARIATION DURING RAIN

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**Abstract:** The power observed on several point-to-point links during rain appears to fluctuate. This work shows that the fluctuations are probably due to the inhomogeneous density distributions of rain drops that move across the line of sight during the sampling time.

## I. INTRODUCTION

This study is aimed at explaining the observed fluctuations in power  $P(t)$  of a 37.5 GHz signal propagated from a transmitter over a 265 m and a 605 meter one-way path to a receiver in the presence of rain [1]. The rain intensity (in mm/hr) was measured by gathering rain drops at one intermediate location. An averaged parameter  $K$  was distilled from power measurements every 0.02 second during a two minute sampling period [1]:

$$K = 10 \log_{10} \frac{\sqrt{\mu_p^2 - \sigma_p^2}}{\mu_p - \sqrt{\mu_p^2 - \sigma_p^2}} \quad (1)$$

where  $\mu_p$  is the average received signal power, and  $\sigma_p^2$  is the variance of the received power around the average. This particular parameter  $K$  is relevant for statistics that obey a Rician distribution in which case  $K = 10 \log_{10}(P_{coh}/P_{inc})$  in terms of coherent power,  $P_{coh}$ , and incoherent power,  $P_{inc}$  [1, 2]. The present results, however, finds the signal statistics to be governed by a lognormal distribution.

The observed results are gathered in Table 1. Several conclusions have been reached. We believe the variations in power appear to be due to local variations of rain drop densities over distances considerably less than the path length. These variations can drift across the line of sight well within the sampling time, and thus cause variations in power. We have rejected other hypotheses. For example, although a multipath reflection from a nearby roof appeared to be able to yield a side-lobe component, this component would not have been incoherent. Off-axis scattering into the receiver from rain drops also has been investigated [3] and rejected because the contribution to  $P_{inc}$  would be too small due to the highly directional receiver antenna used in the measurements (the receiver had a half-power beamwidth of  $1.5^\circ$ ).

Table 1: Measured  $K$  in dB vs Rain Rate  $u_r$ .

$u_r$ (mm/hr)	7.6	7.6	30.5	39.6	45.7	45.7	122	213
265 m		16.8	15.1			14.4	12.4	8.2
605 m	16.6			15.6	15.5			

## II. SIGNAL ANALYSIS

The received signal is well represented by an electric field vector,

$$\vec{E} = \vec{E}_0 e^{-\psi} \quad \text{with} \quad \psi = \gamma - j\phi \quad (2)$$

Here,  $\vec{E}_0$  is the field that would have been received in the absence of rain,  $\phi$  is the phase change due to the presence of rain, and  $\gamma$  is the attenuation due to scattering and absorption of energy out of the line of sight. Hence the received power is  $P = P_0 e^{-2\gamma}$  where  $P_0 = |\vec{E}_0|^2$ . We will argue in the next section that the

attenuation  $\gamma$  can be written as the sum of a steady component  $\bar{\gamma}$  and a Gaussian random part  $\delta\gamma$  with zero mean. Consequently we obtain,

$$\mu_P = P_0 \langle e^{-2\gamma} \rangle = P_0 e^{-2\bar{\gamma}} \langle e^{-2\delta\gamma} \rangle = P_0 e^{-2\bar{\gamma}} e^{2\langle \delta\gamma^2 \rangle} \quad (3)$$

The last step follows from a general property of a lognormal variable with zero mean. The signal variance is

$$\sigma_P^2 = \mu_P^2 (e^{4\langle \delta\gamma^2 \rangle} - 1) \quad (4)$$

Substituting (3) and (4) into (1), we have

$$K = 10 \log_{10} \frac{\sqrt{2 - e^{4\langle \delta\gamma^2 \rangle}}}{1 - \sqrt{2 - e^{4\langle \delta\gamma^2 \rangle}}} \quad (5)$$

### III. VARIATIONS IN ATTENUATION $\gamma$

The attenuation is given [4] by the real part of

$$\psi = \frac{2\pi}{k} \int_0^L dz \int_0^\infty dD n_4(z, D) f(D, k, \epsilon) \quad (6)$$

Here  $k = \omega\sqrt{\mu_0\epsilon}$  is the wavenumber in the medium between  $z = 0$  and  $z = L$ . It can be taken to be the free-space value within the accuracy of this work. The dielectric permittivity of a drop is given by  $\epsilon$ . The particle drop-size distribution  $n_4$  (given in  $m^{-4}$ ) is generally a function of position  $z$  and drop diameter  $D$ . The function  $f(D, k, \epsilon)$  is the forward-scattering amplitude (given in m) of a single spherical particle with effective diameter  $D$ . It may be possible to expand the analysis to non-spherical particles but that is not essential here. We shall make one other assumption to simplify the analysis: the particle drop-size distribution function is given by

$$n_4(z, D) = n_3(z) p(D) \quad (7)$$

where  $n_3(z)$  is the particle density in  $m^{-3}$  and  $p(D)$  is a probability density in  $m^{-1}$ .

Several distributions are commonly used. Among these is the Marshall-Palmer (M-P) distribution [4] with

$$p(z, D) = \Lambda e^{-\Lambda D}, \quad \Lambda = 4100 u_r^{-0.21} m^{-1} \quad (8)$$

given that  $u_r$  is the overall rain rate in mm/hour, and  $D$  is given in m. The M-P connection between density and parameter  $\Lambda$  is  $\Lambda n_3(z) = 8.1 \times 10^6 m^{-4}$ . It follows that  $n_3(z)$  is a function of location  $z$ . As a result,

$$\gamma = \frac{2\pi}{k} F(k, \bar{D}, \epsilon) \int_0^L dz n_3(z) \quad \text{with} \quad F(k, \bar{D}, \epsilon) \equiv \text{Re} \left[ \int_0^\infty dD p(D) f(D, k, \epsilon) \right] \quad (9)$$

Here,  $\bar{D}$  is the mean particle diameter. Equation (9) is the basis for our analysis. The key issue here is the appearance of a path integral over particle density.

Figure 1 illustrates the situation as we hypothesize it to be. The contours are those of the average density  $\bar{n}_3$ . Possibly, higher densities are inside, lower ones outside. The arrows indicate wind direction.

Thus  $n_3(z) = \bar{n}_3 + \delta n_3(z)$  which defines the fluctuation of the density around the average,  $\delta n_3(z)$ . Consequently,

$$\delta\gamma = \frac{2\pi}{k} F(k, \bar{D}, \epsilon) \int_0^L dz \delta n_3(z) \quad (10)$$

can be considered to be a Gaussian variable with zero mean, due to the central-limit theorem of statistics (if there are a sufficient number of small fluctuation cells crossing the line of sight at any time). Thus,

$$\langle \delta\gamma^2 \rangle = \frac{(2\pi)^2}{k^2} F^2(k, \bar{D}, \epsilon) \int_0^L dz_1 \int_0^L dz_2 \langle \delta n_3(z_1) \delta n_3(z_2) \rangle \quad (11)$$

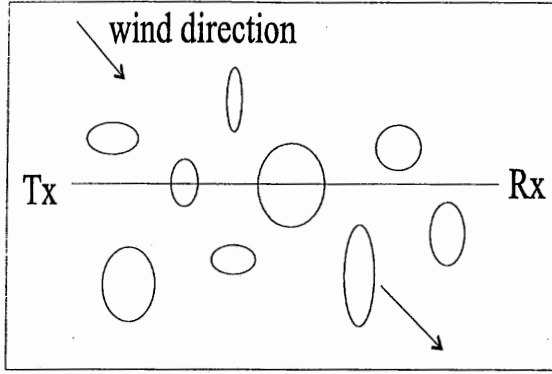


Figure 1: Patches of Rain Across the LOS

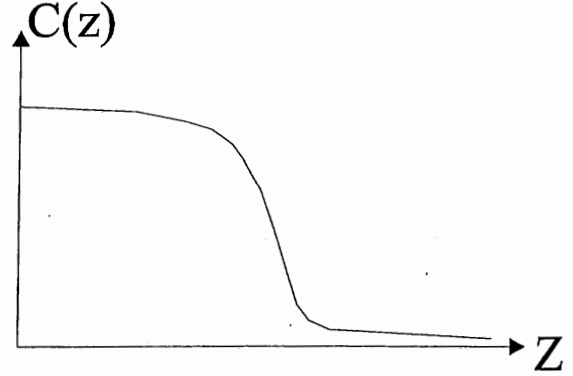


Figure 2: General form  $C(z)$

Under the assumption of isotropy and of a slowly-varying variance, we may set

$$\langle \delta n_3(z_1) \delta n_3(z_2) \rangle = \langle \delta n_3^2(z) \rangle C(|z_1 - z_2|) \quad (12)$$

in which  $C(z)$  has the general form shown in Fig. 2.

A crucial property of this normalized correlation function is  $\int_0^\infty dz C(z) = l$ . In fact,  $l$  is basically a statistical measure of a diameter of a fluctuation  $\delta n_3(z)$ . Under the reasonable assumption that  $l \ll L$  (just how much smaller is somewhat uncertain), we can write,

$$\int_0^L dz_1 \int_0^L dz_2 \langle \delta n_3(z_1) \delta n_3(z_2) \rangle \approx \int_0^L dz_1 \langle \delta n_3^2(z_1) \rangle \int_{-\infty}^{\infty} d\Delta z C(|\Delta z|) \approx 2Ll \langle \delta n_3^2 \rangle \quad (13)$$

Consequently we obtain,

$$\langle \delta \gamma^2 \rangle = \frac{(2\pi)^2}{k^2} F^2(k, \bar{D}, \epsilon) 2Ll \langle \delta n_3^2 \rangle \quad (14)$$

If this is compared to the average of (9), we obtain,

$$\frac{\langle \delta \gamma^2 \rangle}{\bar{\gamma}^2} = \frac{2l \langle \delta n_3^2 \rangle}{L \bar{n}_3^2} \quad (15)$$

$$\langle \delta \gamma^2 \rangle = \bar{\gamma}^2 \beta \quad \text{with} \quad \beta = \frac{2l \langle \delta n_3^2 \rangle}{L \bar{n}_3^2} \quad (16)$$

#### IV. SIGNAL ANALYSIS (CONTD.)

Returning to (5) we note, using (16), that

$$K = 10 \log_{10} \frac{\sqrt{2 - e^{4\beta\bar{\gamma}^2}}}{1 - \sqrt{2 - e^{4\beta\bar{\gamma}^2}}} \quad (17)$$

In order to evaluate (17), we need to have an expression for  $\bar{\gamma}$  as a function of rain rate  $u_r$ , and an estimate of parameter  $\beta$ .

From Olsen, et al. [5], we find that at 37.5 GHz and  $L = 1$  km,

$$8.686\bar{\gamma} \approx 0.260u_r^{1.00} \quad (18)$$

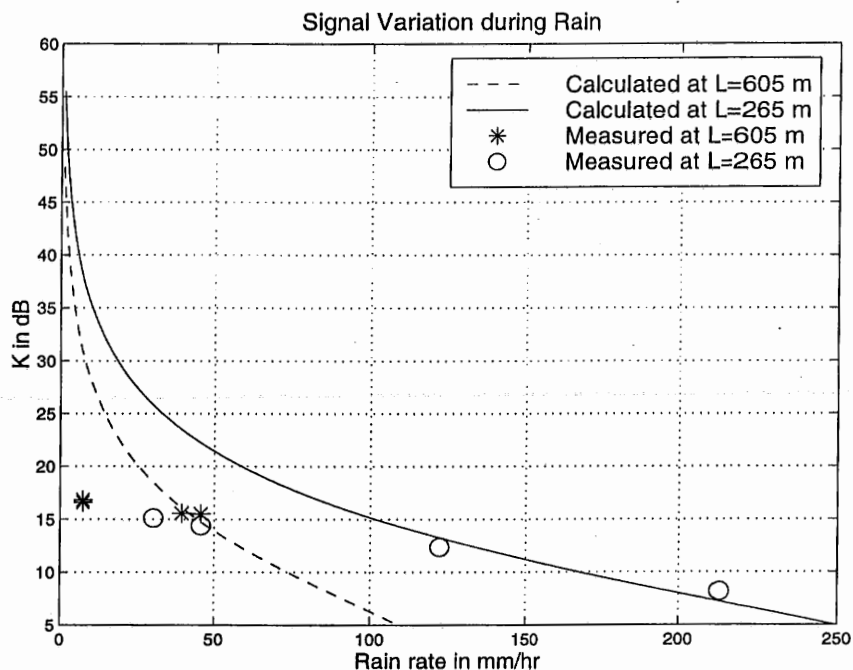


Figure 3: Parameter  $K$  as Function of Rain Rate

Hence at 37.5 GHz and temperature between  $0^\circ$  and  $20^\circ$  C we have:

$$8.686\bar{\gamma} \approx 0.158u_r \quad \text{at } L=0.605 \text{ km} \quad (19)$$

$$8.686\bar{\gamma} \approx 0.068u_r \quad \text{at } L=0.265 \text{ km} \quad (20)$$

For the value  $\beta \approx 0.02$ , the estimated  $K$  based on (17) is plotted in Fig. 3.

Note that the choice of a constant value for  $\beta$  yields a curve with a slope that appears to fit the data if the constant is chosen correctly. The rationale for choosing  $\beta \approx 0.02$  is that this choice appears to give a curve that fits the data. One can only say, in the absence of measurements of  $\langle \delta n_3^2 \rangle$  and of  $l$  that the value of  $\beta \approx 0.02$  is consistent with realistic possibilities. This relatively small value at least indicates that the presence even of relatively small fluctuations of  $\delta n_3$  suffices to yield a  $P_{inc}$  consistent with the observations. Unfortunately, no measurements were taken that might confirm this possibility quantitatively.

## REFERENCES

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